



## **A STUDY OF HEURISTIC BASED ALGORITHMS FOR MATHEMATICAL MODELLING OF DEVELOPMENT AND PERFORMANCE**

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### **ABSTRACT**

Scheduling is described as the process of allocating a number of tasks to a new line using limited resources in order to meet endorsed objectives. brand-new Scheduling in manufacturing is defined as the process of allocating n newline jobs to the available m machines in order to achieve time-based newline objectives such as new line minimising the make span, tardiness, lateness, due date, and so on, as well as cost-based newline objectives such as production cost, transportation cost, and so on. Since manufacturing newline industries play such an important role in a country's economy, the construction of an effective scheduling system to boost growth rates and newline productivity becomes a top priority. This study aims to create an effective newline scheduling method as well as a mathematical model for multi-objective subtask newline scheduling problems in manufacturing. In newline, the multi-objectives are considered. This research includes minimising load balance and cost for industrial robots, newline minimising total weighted completion time for customer order scheduling, and newline developing a good mathematical model for the customer order scheduling issue.

### **Keywords:**

*Artificial Immune System,  
Particle Swam  
Optimization, Subtask  
newline Scheduling  
Algorithm.*



## INTRODUCTION

The increasing complexity in manufacturing industry and growing interest in bio immune and Meta heuristic-based technologies for solving multi-objective subtask scheduling problems has driven interest for taking up this research. It is good to give proper description about the general representation and mathematical model of the proposed heuristic-based algorithms. This chapter explains basic concepts working mechanism, mathematical representation and the applications of heuristic-based algorithms such as Artificial Immune System (AIS), Particle Swam Optimization (PSO), Subtask Scheduling Algorithm (SSA) and Fuzzy based min-max rule algorithm.

### ARTIFICIAL IMMUNE SYSTEM (AIS)

AIS is an artificial intelligence technique that has been used to solve scheduling problems for over ten years. To solve problems, AIS is inspired by vertebrate immunology theory, immune functions, working principles, and immune system mechanisms. It complements existing techniques for pattern recognition, design, modeling, and control by acting as a powerful tool. Optimization, device and network protection, scheduling, data processing and mining, and fault and anomaly detection are the major application domains of AIS. In AIS, the learning process is based on interactions between antibody and antigen populations, which results in a special self-organizing network structure. Each AIS corresponds to a distinct number of potentials in the solutions presented. Other essential features of AIS include the ability to produce innovative solutions in a short period of time, inbuilt memory management, robust recognition, and self-tolerance. Because of these outstanding features, researchers have begun to use AIS to solve multi-objective scheduling problems. The main aim of any scheduling problem is to find a schedule that reduces the application's overall completion time (makespan). Each solution is an antibody (Ab); a number of libraries will be generated, each containing a number of genetic strings that are part of scheduling problem solutions. An antibody (schedule) is generated by concatenating threads from individual libraries.

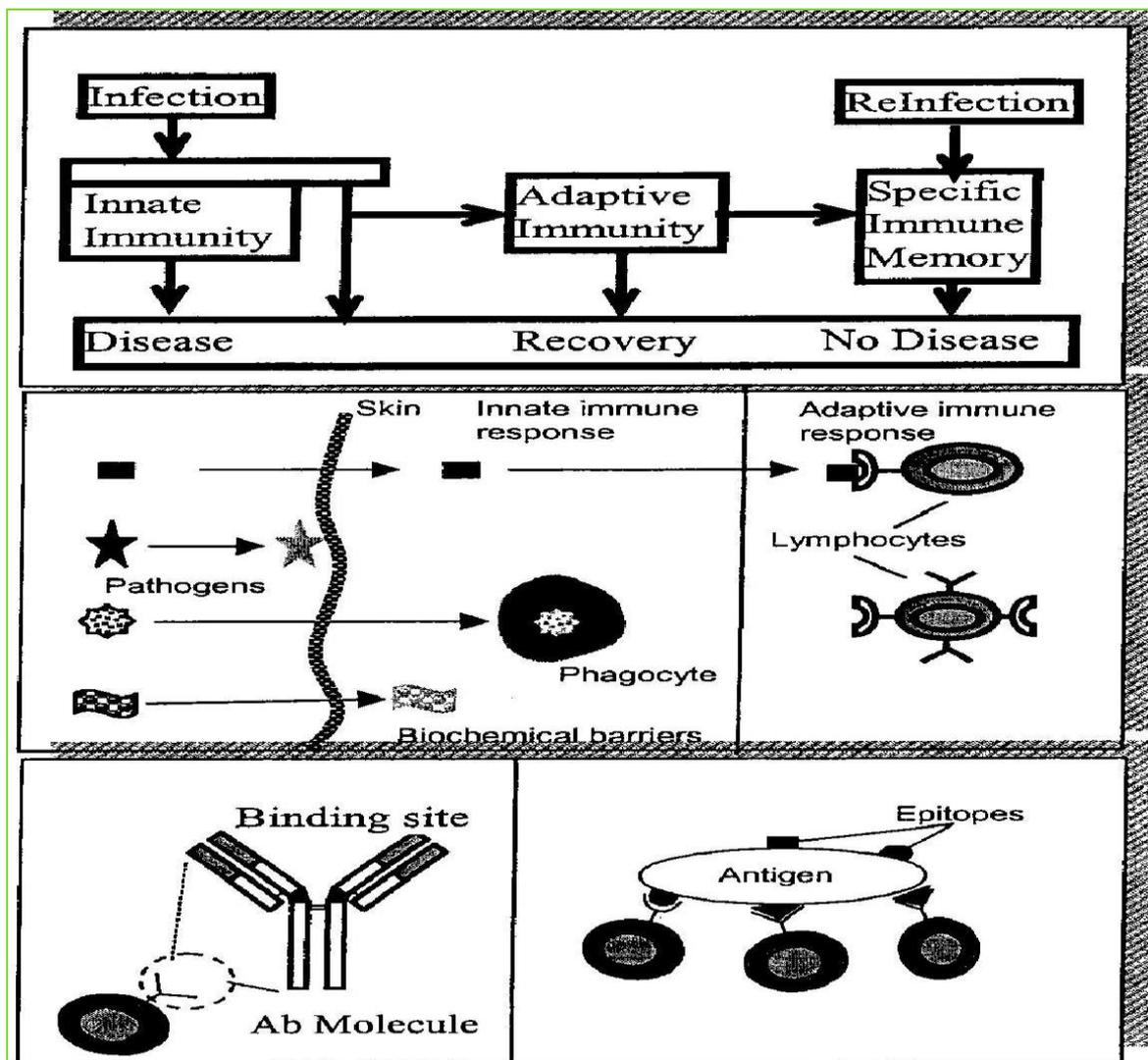
The best specific found was cloned thousands of times. The clones were swapped out, and the best clone was chosen as the solution to the problem.



The Immune System (IS) is a built-in defensive mechanism that serves as a shield to defend all living things from external threats. IS has a biological nature that is resilient, adaptable, and capable of dealing with a wide range of disruptions and uncertainties. The innate and adaptive immune systems are the two main components of this system. The innate consists of organs that protect the body from external attacks, such as skin and mucus. Phagocytes found in the body's tissues and blood assist these protective organs. Phagocytes often send out a signal when they detect a threat. All of these defensive components are responses to a broad category of problems with a limited and predetermined set of responses. If the innate defence elements fail to respond to threats, adaptive is called to respond to the infectious agent. White blood cells (lymphocytes), which are produced by the bone marrow, are the most important component of this mechanism. B-cells, which are produced in the bone marrow, and T-cells, which are produced in the thymus, are the two types of lymphocytes. Antibodies are produced by B-lymphocytes, and some of them live on as memory cells. T-lymphocytes interact with other cells to survive. T-cells are classified into two types: helper T-cells that activate B-cells and destroyer T-cells that kill intracellular pathogens. B-cells that have been stimulated send antigen pieces to destroyer T-cells. Immune recognition takes place between the receptor's area and an epitope. Antibodies bind to several molecules on the infectious agent's surface rather than the entire infectious agent. The immune system's mechanism is depicted in Figure 1.

Antibodies are two-part molecules with variable and constant regions. The IS has the ability to speed up the adaptation process due to variations in the variable field. According to research on antibody diversity produced during the immune response, the number of somatic mutations in the variable region increases with time. This raises the antibody's affinity for the antigen. Mutation levels at high levels play an important role in IS maturation. Actually, the hyper mutation mechanism is totally random; several antigens will be destroyed during the mutation process. The best way for the IS to solve this problem is to increase the population of high-affinity antibodies. As a result, selection plays a crucial role in the formation of formative high affinity antibodies.

There are a plethora of theoretical models focused on the IS mechanism. Bone-marrow models, Negative selection models, Clonal Selection dependent algorithm models, and others are the most commonly used models. The selection of these models is entirely dependent on the existence of a problem. To find the best solution to our research dilemma, we used a Clonal selection-based algorithm. The following is a thorough summary of the same model.

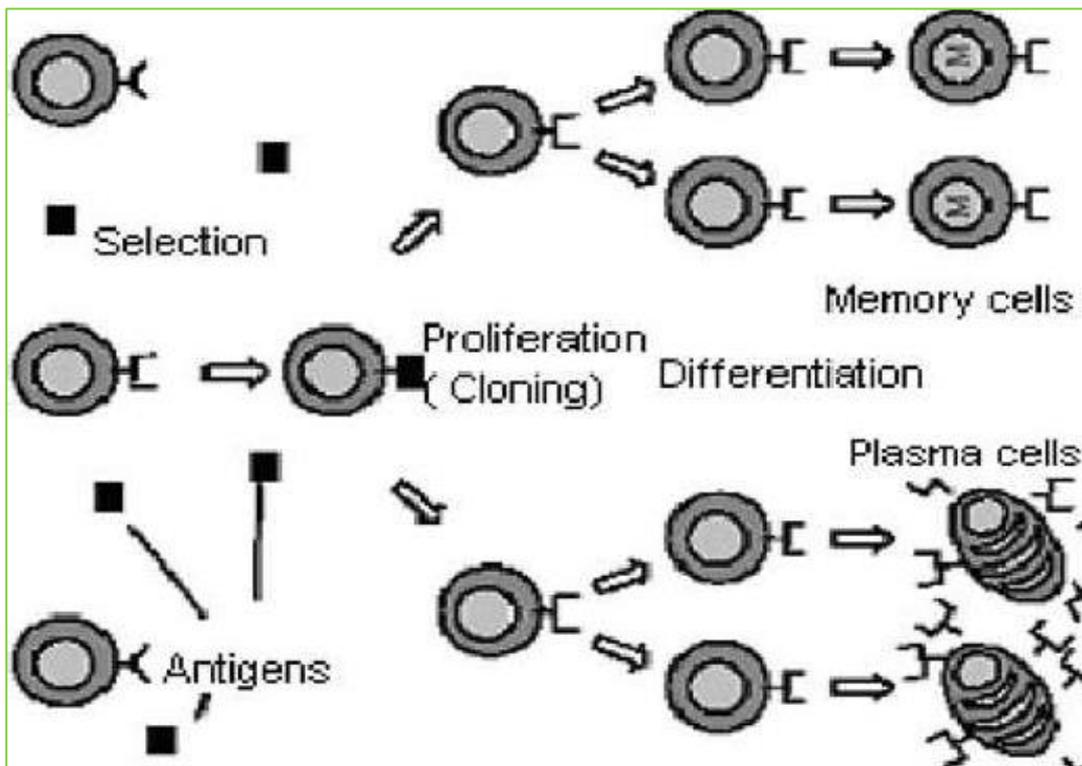


**Figure – 1: IS Function Flow (Top), Defense in The Immune System (Middle) and Immune Recognition (Bottom)**

### CLONAL SELECTION PRINCIPLE

The Clonal selection principle is based on antigen determined affinity maturation process of B-cells and associated hyper mutation mechanism. De Castro et al. [26] discussed two important principles of affinity maturation in B-cells. The first principle states that proliferation of B-cells is straight relational to the similarity of the antigen that binds it, representative developed the attraction, and more clones are produced. Second opinion states that the mutation under went by the antibody of a B-cell is contrary wise relative to the attraction of the antigen it binds. Using these two principles they developed one of the broadly used Clonal selection based AIS algorithm called CLONALG (Figure-2).

In the development of Clonal selection based algorithm the following aspects were considered; collection and cloning of efficient stimulated cells, affinity mutation and recollection of the clones with high affinity, hyper mutation of cells proportional to their affinity, preservation of the memory cells, decease of non-stimulated cells, production and preservation of diversity.



**Figure – 2: Clonal Selection Principle**



In the algorithm, antibody is a entrant agenda and antigen is a participant agenda which will be the best agenda till that time prompt created by the algorithm. The stepwise working procedure of the algorithm is given below.

- Step 1: Generate initial population of antibodies that is generation of potential schedules.
- Step 2: Obtain the affinity of the generated antibodies.
- Step 3: Generate clones for each antibody; calculation of number clones to be generated depends on the affinity of the respective antibody.
- Step 4: Generate matured cloned population by hyper mutation process.
- Step 5: Choose the best clone and discard the remaining clones.
- Step 6: Considered the new population as the candidate for the next generation.

This mechanism is continued till the best solution is obtained. This is the best schedule for the optimal problem considered, meeting the pre-defined constraints.

### **PARTICLE SWAM OPTIMIZATION (PSO)**

Particle Swarm Optimization (PSO) is an evolutionary population based intelligent technique inspired by the social activities of animals like flock of birds, school of fish etc. PSO technique was proposed. The remarkable features of PSO made the researchers to focus more on this technique are namely optimization through social evolution, its simplicity of usage, it requires simple mathematical operators, and it is low-cost in terms of both memory requirements and time has. Here are few applications that have implemented PSO technique are; chemical engineering, data mining, the voltage control problem, environmental engineering, pattern recognition, to solve Scheduling problems and task allocation.

The PSO algorithm is analogous to the existing EA. In PSO, the number of particles represents population in a problem space. Particles are randomly initialized. All particles possess fitness values which are calculated by a fitness function, this has to be optimized in every generation. Each particle has its best position denoted as pbest, is the best fitness value (result) reached by the particle. The

best position among the total group of particles denoted by gbest, is the best particle fitness in a total population. The size of the population is problem dependent the most general sizes considered are 20–50. In each generation the velocity and particles position will be calculated using Equation 1 and 2, respectively. In Equation 1 the first part is the inertia of the prior velocity. The second part is called cognition part, indicates individual thinking, and the last part is social consciousness, indicates cooperation between the particles. The particle new position is obtained by adding up new velocity to the existing position according to Equation 2.

$$V_{i,k+1} = wV_{i,k} + (rand)_1 C_1 (P_i - X_{i,k}) + (rand)_2 C_2 (P_g - X_{i,k}) \quad (1)$$

$$X_{i,k+1} = X_{i,k} + V_{i,k+1} \quad (2)$$

Where,

$$\left[ \begin{array}{l} "i = i^{th} \text{particle} \\ X_{i,k} = \text{position of particle 'i' in iteration 't'} \\ V_{i,k} = \text{velocity of particle 'i' in iteration 't'} \\ p_i = \text{previous best position of particle 'i'} \\ P_g = \text{previous best position among all the particles } (g_{best}) \\ w = \text{inertial weight} - \text{balances local and global exploitations of the particles} \\ C_1 \text{ and } C_2 = \text{learning factors} - \text{controls the influence of } p_{best} \text{ and } g_{best} \text{ on search process} \\ (rand)^1 \text{ and } (rand)^2 = \text{random numbers} - \text{taken within the range } [0,1] \end{array} \right.$$

The summarized process for standard PSO is as follows:

Step 1: Initialize randomly a population of all particles with their random positions and velocities.

Step 2: Estimate the suitability ideals of all elements, set gbest of each unit like to its existing place, and set gbest identical to the place of the best original element.

Step 3: Calculate the velocity and position of each particles using Eqs. (1) and (2).

Step 4: calculate the fitness values of each particles and compare its existing fitness value with its pbest value. If the existing value is better, then update pbest with the existing position and fitness value.



Step 5: Find out the best particle of the existing population with the best fitness value. If the fitness value is better than gbest, then update gbest with the existing best particle.

Step 6: If maximum number of iterations is met, then output gbest and its fitness value; if not, go to Step3.

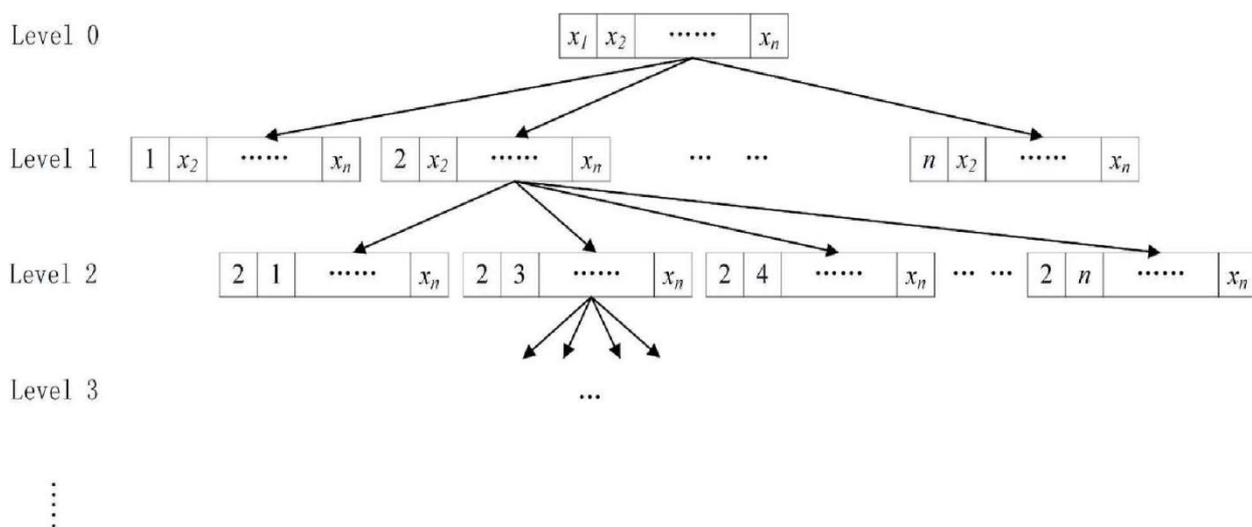
### **SUBTASK SCHEDULING ALGORITHM**

Customers' preferences and specifications are complex as a result of mass customization. As a result, it is more prudent to concentrate on multi-task scheduling in order to have the best services and please customers. Multi-tasks can be classified into two categories: homogeneous and heterogeneous multi-tasks. The homogeneous task has identical characteristics and can be completed using the same manufacturing processes. When working on a heterogeneous mission, the features are not all the same and must be processed through a variety of processes. Decomposing a task into a variety of subtasks and processing them by aggregated distributed resources is the best way to manage heterogeneous multi-tasking (Figure-3).

Task decomposition is the method of dividing a large piece of work into two or more smaller tasks known as subtasks. The working organisation of insect societies served as a general inspiration for this. Many animals, such as ants, bees, termites, and wasps, engage in mission decomposition. Almost all of the insects listed above are foragers. Time and money are factors in task decomposition. Profit comes from either improving individual performance or improving the overall system. The honey bee's accumulation of nectar is the best example of mission decomposition. Foragers carry nectar to bees working inside the nest, known as receivers, who then deposit the nectar into cells. As a result, nectar selection is decomposed among foragers and receivers. Task decomposition causes a novel feature: transportation between forager and recipient, by dividing the task – nectar selection – into two subtasks such as foraging and receiving. The resources required to link each subtask in a decomposed mission. Linking related tasks is required for task decomposition. A task is a discrete piece of work that needs to be completed. Foraging encompasses the collection, repossession, and storage of forage as a whole. If forage is gathered and taken halfway to its storage location, the job is clearly incomplete. The complete completion of work is, in general, dependent on



a number of tasks. Due to the heterogeneous relationship between subtasks in a process, several structures are created when a task is decomposed into a number of subtasks. These structures are converted into tree structures, which offer only aggregation relationships and make it simple to explain the types of process. Decomposition of tasks is a relatively recent concept. The decomposition of a task into subtasks and their distribution among available resources are critical because they have a major impact on output.



**Figure -3 Decomposition of A Task into Number of Subtask**

A number of articles are published in recent years to advance the effectiveness and usefulness of task decomposition methods by Pi et al. [69]. The detailed description about subtask scheduling algorithm is discussed in chapter 6, here we will give basic concept followed in developing subtask scheduling algorithm.

**BASIC CONCEPT OF SUBTASK SCHEDULING (TASK DECOMPOSITION)**

- 1) **Partitioning:** The existing most promising task is decomposed into number of subtask. In each approximation and the left-over solutions is aggregated into the nearby task.



**Random Sampling:** A number of samples are chosen randomly from each subtask and the nearby task. Depending on the problem different schemes for sampling can be used. However, the probability of selecting each solution must be positive and when sampling high quality solutions are preferred.

- 2) **Evaluation of the Promising Index:** The samples objective function values are evaluated for each subtask and nearby task, based on these values the promising indices are determined.
- 3) **Backtracking:** Based on the promising indices of the subtask and the nearby task, the most promising task in the subsequent approximation is determined. If the most promising index corresponds to a subtask, then that subtask becomes the most promising task in the next approximation. If the most promising index corresponds to the nearby task, then backtracking is performed.

### **FUZZY BASED MIN-MAX RULE ALGORITHM**

Scheduling is a process of assigning limited resources within the constraints to meet the well-defined objectives. A feasible scheduling satisfies the constraints associated with set of tasks and available resources. In a multi-processor scheduling environment, researchers find greater challenges in order to obtain feasible scheduling. Many techniques are proposed to address these challenges like AIS algorithms (as we discussed in the previous section) and Fuzzy techniques. In this section we are going to discuss scheduling problem using fuzzy technique.

Fuzzy logic was proposed by Zadeh [98] and has applications in the field of target detection, image analysis, computer science and intelligent information processing etc. fuzzy logic is an substitute to Boolean logic, in which degree of truth value is used to observe the mode of reasoning, which plays an vital role in decision making ability in uncertainty and imprecision environment. Fuzzy inference consists of three stages as follows:



1. **Input Stage:** Receives input such as target line, completing time, response time etc. and maps these inputs to suitable membership functions.
2. **Processing Stage:** Each suitable rule is invoked and the corresponding results are produced and are joined to give an input to the output stage.
3. **Output Stage:** Converts the joined result back into a specific value.

The membership function depicts curve which enables us to know how each inputs mapped to a membership value between 0 and 1. The processing stage is based on number of logical rules. The logical rules are stated in the form of IF – THEN rules (in detail if- then rules are discussed in the next session). In order to understand the fuzzy inference technique, it is more relevant to know some basic definitions of fuzzy concept which are given below:

## BASIC DEFINITIONS

### Fuzzy set:

Fuzzy set is expressed as a Membership function  $\mu_A$  which maps all the members in universal set  $Y$  to  $\{0, 1\}$ . Denoted as  $\mu_A : Y \rightarrow \{0,1\}$

In fuzzy sets, each one of the element is mapped to  $[0, 1]$  by membership function  $\mu_A : Y \rightarrow [0,1]$ ,  $[0, 1]$  is the real numbers between 0 and 1 including 0 and 1.

### Operations on Fuzzy set:

Among various operators, the standard operators are complement, Maximum (Max) and Minimum (Min) are the fundamental and simple operators. A range of fuzzy theories are developed based on these operators.

Complement set  $A$ , union  $A \cup B$  and intersection  $A \cap B$  are the standard operations of fuzzy theory and are defined as,

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

$$\mu_{A \cup B}(x) = \text{Max} [\mu_A(x), \mu_B(x)]$$

$$\mu_{A \cap B}(x) = \text{Min} [\mu_A(x), \mu_B(x)]$$



### **Fuzzy Rule Representation-Inference**

Inference is defined as a process of obtaining new information by using existing knowledge. The most popular inference used is of the form “if-then”. Illustrated as,

“If x is a, then y is b.”

The rule is interpreted as an implication and consists of the antecedent (if part) and consequent (then part).

Example: “x is a” then we can infer and obtain new result, “y is b”.

### **Fuzzy if-then Rules**

A fuzzy rule generally assumes the form R: “If x is A, then y is B”.

Where A and B are linguistic values defined by fuzzy sets on universe of discourse X and Y, respectively. The rule is also called a fuzzy implication or fuzzy conditional statement. The part x is A is called the “antecedent” or premise, while y is B is called the consequence or conclusion. Before we employ fuzzy if-then rules to represent and analyze a system, let’s formalize the meaning of the expression R: “If x is A then y is B”,

This is sometimes abbreviated as  $R : A - B$

In essence, the expression describes a relation between two variables x and y. This suggests that a fuzzy rule can be defined as a binary relation R on the product space  $X \times Y$ .

### **Fuzzification**

Transforms input values into fuzzy values and performs the following functions.

- Receives the input values
- Transforms input values of variable into corresponding universe of discourse
- Transforms input statistics into fuzzy sets.

### **Defuzzification**

In many applications, a control command is given as a crisp value. Hence it is required to defuzzify the outcome of the fuzzy inference. A defuzzification is a process used to obtain a non- fuzzy control action that best represents the possibility distribution of a fuzzy control action. Unfortunately, there is



no systematic procedure for selecting a good defuzzification strategy, and thus the selection process depends on the properties of application.

### Min-Max Algorithm

This principle is based on fuzzy set theory that provides a common method for extending crisp domains of mathematical expressions to fuzzy domains. This method generalizes an ordinary mapping of a function to a mapping between fuzzy sets.

Suppose that  $g$  is a function from  $X$  to  $Y$ , and  $A$  is a fuzzy set on  $X$  defined as

$$A = \{(x_1, \mu_A(x_1)), (x_2, \mu_A(x_2)), \dots, (x_n, \mu_A(x_n))\}$$

Then the principle states that the Composition of fuzzy relations  $R$  and  $S$

$$SR = SoR \{(x, y), \mu_{SR}(x, z)\}$$

Where  $\mu_{SR}(x, z) = \min \max \{(\mu_R(x, y)), (\mu_S(y, z))\}$

### CONCLUSION

We discussed the basic concept and working mechanism of heuristic-based algorithms such as AIS, PSO, and SSA, as well as the fuzzy concept in scheduling, in this research paper. Many researchers have been drawn to AIS because of its ability to produce new solutions in a short period of time, as well as its robust identification and self-tolerance. Similarly, the PSO algorithm has advantages due to its ease of use and the fact that it only includes basic mathematical operators. The subtask scheduling algorithm is useful in the case of heterogeneous multiple tasks with different scheduling characteristics that must be processed by a variety of processes. In order to obtain feasible scheduling, fuzzy based algorithms are applied in a multi-processor scheduling system, where researchers face greater challenges. Algorithms can be chosen and applied based on the problem's goal and the design of the scheduling.



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